4.5 Linearizations and differentials

$L(x) = f'(a)(x-a) + f(a)$

$y = m(x-x_1)+y_1$ } equation of tangent line at $x=a$

Find the linearization of $f(x)=\sqrt{1+x}$ $(1+x)^{1/2}$ at $x=0$ easy

Use the linearization to approximate $\sqrt{1.02}$ $x=.02$ hard

Use a calculator to determine the accuracy of the approximation.

$f'(x)=\frac{1}{2}(1+x)^{-\frac{1}{2}}$

$f'(x)=\frac{1}{2\sqrt{1+x}}$

$f'(a)=f'(0)=\frac{1}{2}$

$a=0$

$f(a)=f(0)=\sqrt{1+0}=1$

$L(x)=\frac{1}{2}(x-0)+1=\frac{x}{2}+1$

$f(.02)\approx L(.02)$

$\sqrt{1.02}\approx\frac{.02}{2}+1=.01+1=1.01$

$1.00995\approx 1.01$

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Use Linearizations to approximate $\sqrt{123}$

$f(x)=\sqrt{x}$ $a=121$

$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

$f'(a)=\frac{1}{2\sqrt{121}}=\frac{1}{2\cdot 11}=\frac{1}{22}$

$f(a)=\sqrt{121}=11$

$L(x)=\frac{1}{22}(x-121)+11$

$L(123)=\frac{1}{22}(123-121)+11$

$=\frac{1}{22}\cdot 2+11=11\frac{1}{11}$

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Differentials

$dy = f'(x)\,dx$

$\frac{dy}{dx}=f'(x)$ derivative or ratio of 2 differentials

Given $A=\pi r^2$ find the differential dA and evaluate dA for r=10, $dr=0.1$

$dA=2\pi r\cdot dr$

$dr=\Delta r$ $(dx=\Delta x)$

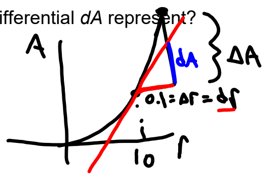
$dA=2\pi\cdot 10\cdot(0.1)$

$dA=2\pi$

$dA\approx\Delta A$ $(dy\approx\Delta y)$ good approximation if dx is small

What does the differential dA represent?

$A=\pi r^2$



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Percentage change is estimated by $\frac{df}{f(a)}*100$

Ex 10. If the radius of the earth is estimated to be 3959 ± 0.1 what effect would the tolerance of 0.1 have on an estimate of the earth's surface area? radius

$dr=0.1$

$S=4\pi r^2$

$\Delta S\approx dS = 8\pi r\cdot dr = 8\pi(3959)(0.1)$

tolerance for surface area (error) $= 9950$ mi^2

percent change $\frac{9950}{4\pi(3959)^2}\times 100\% =$ $.005\%$

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